MMAT5390: Mathematical Image Processing

Assignment 3

Due: 23:59 Wednesday, March 12, 2025

Please give reasons in your solutions unless otherwise specified.

1. We have covered the following property of the Discrete Fourier Transform (DFT) of convolution in class:

   - Given images $g, w \in \mathbb{R}^{M\times N}$, the DFT of their convolution is given by:

     $$\widehat{g * w}(m,n) = MN \cdot \widehat{g}(m,n) \odot \widehat{w}(m,n),$$

     where $\odot$ denotes the pointwise product of two images.

   Question: What about the converse direction? Under the same assumptions, and further assuming $x(m,n) = g(m,n) \cdot w(m,n)$, prove that:

   $$\widehat{x}(k,l) = \sum_{p=0}^{M-1}\sum_{q=0}^{N-1} \widehat{g}(p,q) \cdot \widehat{w}(k-p, l-q).$$

2. Let $g = g(k,l)$ be a $N \times N$ image. Let $\tilde{g}$ be defined as:

   $$\tilde{g}(k,l) = g(k,l) \times e^{2\pi j \frac{m_0 k + n_0 l}{N}}$$

   Prove that

   $$DFT(g)(m - m_0, n - n_0) = DFT(\tilde{g})(m,n)$$